Turbulence Structure in an Initial Mixing Region of a Two-Dimensional Curved Jet

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The turbulent flow structure in an initial mixing region of a two-dimensional curved jet along a circular arc is investigated by measuring the intermittency factor, the turbulence scales, the skewness factor, and flatness factor. The results show that the profiles of the intermittency factor and the integral scale exhibit an obvious asymmetry due to the influence of the centrifugal force. These quantities are clearly enhanced in the outer shear layer of the curved jet and are reduced in the inner shear layer of it. However, downstream of the initial mixing region, the asymmetry in the profile of the intermittency factor decays gradually. On the other hand, the effects of the curvature are hardly observed in the profiles of the microscale, the skewness factor, and the flatness factor, even within the initial mixing region.

Nomenclature

b_0 = slot thickness
$b_{1/2}$ = half the jet thickness based on half the maximum velocity
F = flatness factor = $\overline{u^4}/(\overline{u^2})^2$
q^2 = $u^2 + v^2 + w^2$
$R_{11\tau}$ = auto-correlation coefficient
S = skewness factor = $\overline{u^3}/(\overline{u^2})^{3/2}$
t = time
U = mean velocity component in the x direction
U_c = velocity at the jet centerline
u, v, w = fluctuating velocity components in the x, y, and z directions
x, y, z = coordinate system
$y_{\gamma=0.5}$ = mean location of the interface
γ = intermittency factor
η = $(|y| - b_{1/2})/x$
$\Lambda_{x\tau}$ = integral scale
λ_x = microscale
τ = time

Introduction

RECENTLY, significant efforts are being devoted to improving the aerodynamic and optical characteristics of aerodynamic windows used in high-power gas lasers.[1,2] The aerodynamic windows utilize one or more two-dimensional gas jets to permit the extraction of the laser beam and support the pressure difference between the lasing gas and atmosphere. Due to the presence of the constant pressure difference, the two-dimensional jets are caused to bend along a circular arc. Therefore, the coherence of the laser beam extracted through the aerodynamic window is degraded by the turbulent flow structure of the curved jet. To predict the optical characteristics of the extracted laser beam precisely, the turbulence structure of the curved jet along a circular arc must be understood.[3,4]

Numerous works address the curvature effects on turbulent wall shear layers, but very few study turbulent free shear layers. In the previous paper,[5] therefore, the turbulence structure in an initial mixing region of a two-dimensional curved jet along a circular arc was investigated by measuring the mean velocity, the turbulent intensities, the Reynolds stress, the turbulent energy balance, and the Reynolds stress balance. In the present paper, further details of the turbulence structure, including the intermittency factor, the turbulent scales, the skewness factor, and flatness factor, are presented.

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Experimental Apparatus

The apparatus used in the present investigation is shown schematically in Fig. 1.[5] The apparatus consists of a two-dimensional nozzle, a cubic test chamber, and an exhaust opening. A blow-down air supply system is used to provide the air flow to the nozzle, whose exit thickness b_0 and aspect ratio are 10 mm and 20, respectively. The contraction section of the nozzle consists of the two arcs with the radius of 29 mm and is followed by a constant area section of 40 mm in length. The jet generated by the nozzle is injected into the test chamber and is exhausted through an opening at the downstream wall of the test chamber. The width of the exhaust opening is 50 mm. As shown in Fig. 1, the nozzle exit plane is inclined to the upstream wall of the test chamber with an angle of 30 deg. Therefore, a curved jet confined by the two vertical walls of the test chamber is generated automatically by injecting air into the test chamber continuously.

The coordinate system used in the present paper is also shown in Fig. 1, where x is the coordinate parallel to the jet centerline and y and z are the coordinates perpendicular to x. In the present paper, the jet centerline is defined as the streamline issuing from the center of the nozzle exit. To determine the jet centerline, an electrically heated tungsten wire of 0.2 mm diam can be fixed at the center of the nozzle exit along a

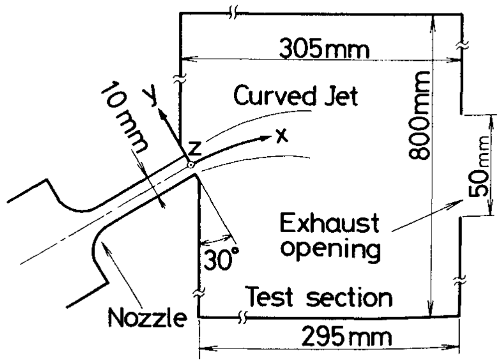

Fig. 1 Experimental apparatus.

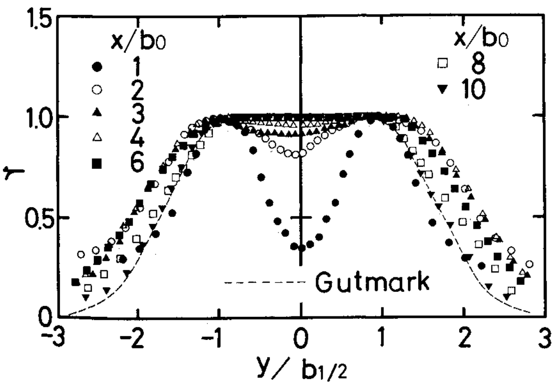

Fig. 2 Profiles of the intermittency factor γ.

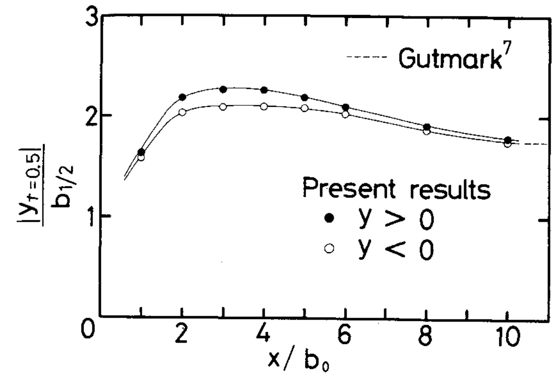

Fig. 4 Growth of the turbulent zone with downstream distance.

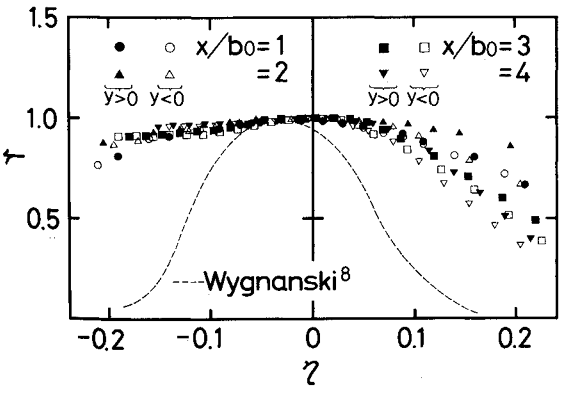

Fig. 3 Profiles of γ plotted using η.

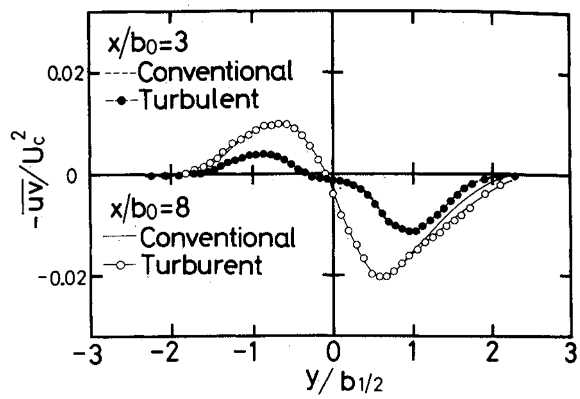

Fig. 5 Profiles of the Reynolds stress.

spanwise line. Then, the path of the streamline starting at the wire can be traced by traversing a thermocouple normal to the heated wake.

The velocity measurements are made with single-wire probes along with constant temperature anemometers and linearizers. The hot wire is platinum-coated tungsten with diameter of 5 μm and active length of 1 mm. The probe is calibrated before and after each run to check for the repeatability of the calibration. The signal is registered in a data recorder, then processed by an analog electronic circuit and/or a micro-computer. The intermittency factor is evaluated by an electronic circuit essentially similar to that suggested by Bradbury.[6] The microscale is derived using the data on the mean velocity, the turbulent intensity, and the dissipation term in the turbulent energy equation shown in the previous paper.[5]

As in the previous paper, the present experiments are carried out with the slot Reynolds number set at 1.7×10^4. Many aerodynamic windows involve high subsonic or supersonic flows. However, the flow of the present study is essentially incompressible, since the jet Mach number is below 0.1. At the nozzle exit, the displacement thickness and momentum thickness of the nozzle wall boundary layers calculated from the measured velocity profiles are about 0.15 mm and 0.08 mm, respectively, and the measured turbulence level is 0.65%. The curved jet confined by two vertical walls of the test chamber is two-dimensional except in the vicinity of the walls. The variations in the mean velocity and the turbulence level across the span do not exceed 1% over 90% of the span width, even at a downstream distance of 20 b_0. As shown in the previous paper,[5] the radius of curvature of the jet centerline is 310 mm, and the potential core disappears about 4 slot thicknesses downstream of the nozzle exit.

Results and Discussion

The measured lateral distributions of the intermittency factor γ are shown in Fig. 2, where $b_{1/2}$ is half the jet thickness based on half the maximum velocity, and compared with the result for a straight jet given by Gutmark et al.[7] The growth of $b_{1/2}$ with downstream distance was given in the previous paper.[5] Gutmark's results were obtained far downstream from the nozzle ($x/b_0 \doteqdot 100$) where the self-preserving state has been attained. The profiles of γ measured in the initial mixing region ($x/b_0 < 4$) have a hollow in the central portion of the jet due to the presence of the potential core. The hollow disappears as soon as the potential core disappears, and the profile of γ at $x/b_0 = 10$ agrees very well with the results of Gutmark et al. The profiles of γ plotted using $\eta = (|y| - b_{1/2})/x$ are shown in Fig. 3 and are compared with the results given by Wygnanski et al.[8] for a simple shear layer. Wygnanski's results were also obtained far downstream of the nozzle where the self-preserving state has been attained. The profiles of γ in the shear layers of the initial mixing region is considerably wider compared with that of the self-preserving simple shear layer. The intermittency factor is very large in the central portion of the jet ($\eta < 0$), since the turbulence develops rapidly in the potential core. This characteristic is in contrast with the fact that the growth of the thicknesses of the outer ($y > 0$) and inner ($y < 0$) shear layers at the extremes of the initial mixing region is not very large, compared with that of the self-preserving simple shear layer as shown in Fig. 4 of the previous paper.[5] The mean locations $y_{\gamma=0.5}$ of the interfaces between the turbulent zone and irrotational zone develop with the downstream distance as shown in Fig. 4. The values of $|y_{\gamma=0.5}|/b_{1/2}$ are nearly one at the nozzle exit and grow very rapidly in the initial mixing region. However, downstream of the initial mixing region, they decrease gradually and approach Gutmark's results.[7] Prandtl suggested that there should be an enhanced mixing in the outer portion of the curved jet and reduced mixing in the inner portion of it due to the influence of centrifugal forces on the parcels of fluid that transfer momentum from layer to layer. Figure 4 shows that the extent of the turbulent zone in the outer portion of the jet is clearly larger than that in the inner portion. This tendency is significant in the initial mixing region. The asymmetry in the

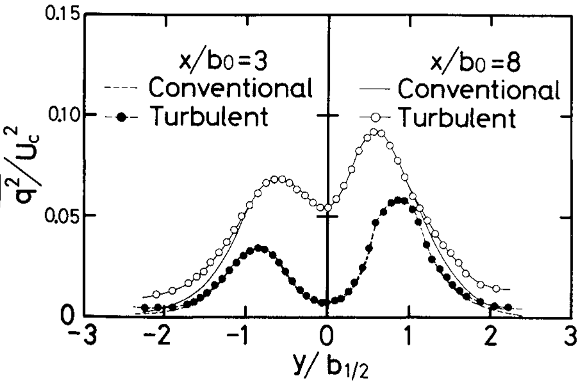

Fig. 6 Profiles of the turbulent energy.

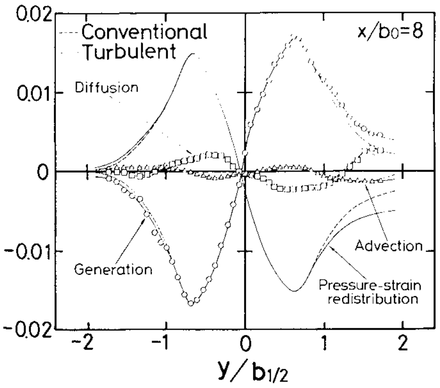

Fig. 7 Reynolds stress balance.

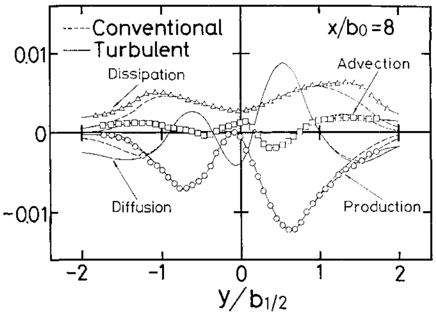

Fig. 8 Turbulent energy balance.

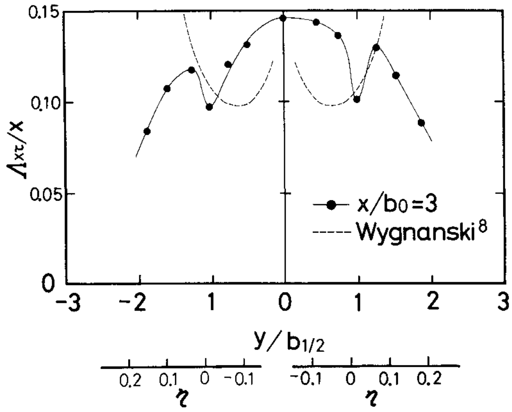

Fig. 9 Profile of $\Lambda_{x\tau}$ at $x/b_0 = 3$.

extent of the turbulent zone is very similar to the asymmetry in the mean velocity profile with regard to the dependence on x. As shown in the previous paper,[5] the thickness of the outer shear layer of the initial mixing region is somewhat larger than that of the inner shear layer, but the difference between the outer and inner shear layer thicknesses decreases rapidly downstream of the initial mixing region.

The distributions of the Reynolds stress and the turbulent energy in the turbulent zone are compared in Figs. 5 and 6, respectively, with the conventional distributions. In Figs. 5 and 6, $q^2 = u^2 + v^2 + w^2$, u, v, and w are fluctuating velocity components in the x, y, and z directions, respectively, and U_c is the velocity at the jet centerline. The conventional distributions of the Reynolds stress and the turbulent energy have been already shown in the previous paper. As pointed out in the previous paper,[5] both the Reynolds stress and the turbulent energy are much larger on the outer edge of the jet, supporting the importance of centrifugal forces. Both quantities are substantially higher at $x/b_0 = 8$ than $x/b_0 = 3$, since the turbulence develops in the downstream distance. The turbulent zone averaged distributions are obtained dividing the conventional value by γ. The Reynolds stress and the turbulent energy in the turbulent zone at $x/b_0 = 8$ are considerably higher than the corresponding conventional values in the outer and inner extremes of the jet. However, the turbulent zone averaged values at $x/b_0 = 3$ are almost identical to the conventional values because the mean location of the interfaces are considerably far from the jet centerline.

The Reynolds stress balance and the turbulent energy balance at $x/b_0 = 8$ are respectively shown in Figs. 7 and 8. The ordinates of these figures represent the terms in the Reynolds stress transport equation and the turbulent energy equation respectively. Each term is made dimensionless by multiplying $b_{1/2}/U_c^3$. The method of evaluation of the terms was described in the previous paper.[5] Figure 7 shows that the turbulent zone averaged values of all terms in the Reynolds stress transport equation are higher than the corresponding conventional values for $|y/b_{1/2}| > 1$. Figure 8 also shows that the turbulent zone averaged values of the production, dissipation, and diffusion in the turbulent energy equation are considerably higher than the corresponding conventional values at the outer and inner extremes of the jet. However, the turbulent zone averaged value of the advection is nearly constant for $|y/b_{1/2}| > 1$ and it is somewhat lower than the conventional value in the region $1.5 > |y/b_{1/2}| > 1$. These trends were also observed by Gutmark et al.[7] for a self-preserving, two-dimensional straight jet. The conventional distributions of the Reynolds stress balance and the turbulent energy balance at $x/b_0 = 3$ were shown in the previous paper.[5] In the initial mixing region, the zone averaged values of the terms in the Reynolds stress transport equation and the turbulent energy equation are almost identical to the conventional values, since the turbulent zone spreads widely.

The auto-correlation coefficient $R_{11\tau}$ at any point of the jet is defined by

$$R_{11\tau} = \frac{\overline{u(t)u(t+\tau)}}{[\overline{u(t)^2}]^{1/2}[\overline{u(t+\tau)^2}]^{1/2}} \tag{1}$$

where t and τ are the time. Using Taylor's hypothesis, the integral scale $\Lambda_{x\tau}$ and microscale λ_x are obtained as follows:

$$\Lambda_{x\tau} = U \int_0^\infty R_{11\tau}(\tau)\, \mathrm{d}\tau \tag{2}$$

$$\lambda_x^2 = 2\overline{u^2}U^2/\overline{(\partial u/\partial t)^2} \tag{3}$$

where U is the mean velocity in the x direction.

Figures 9 and 10, respectively, show the profiles of $\Lambda_{x\tau}$ at $x/b_0 = 3$ and 8, and compared with Wygnanski's result[8] for a self-preserving simple shear layer and Gutmark's result[7] for a self-preserving, two-dimensional straight jet. As shown in Fig. 9, the profile of $\Lambda_{x\tau}$ at $x/b_0 = 3$ takes minimum values in the outer and inner shear layers of the initial mixing region. Wygnanski's results also show the existence of a minimum value of $\Lambda_{x\tau}$ in the simple shear layer. Therefore, in the initial mixing

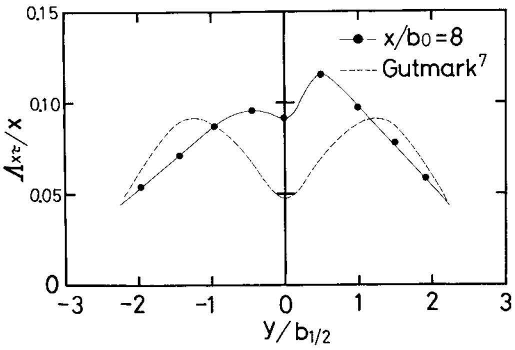

Fig. 10 Profile of $\Lambda_{x\tau}$ at $x/b_0 = 8$.

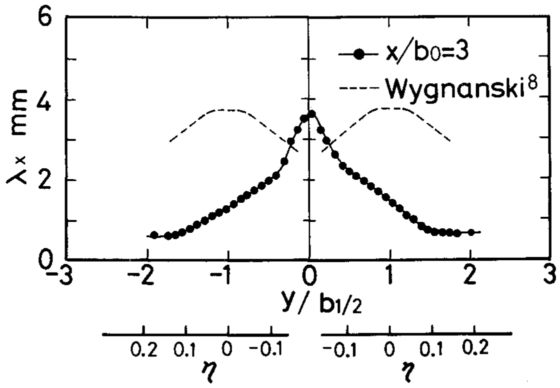

Fig. 11 Profile of λ_x at $x/b_0 = 3$.

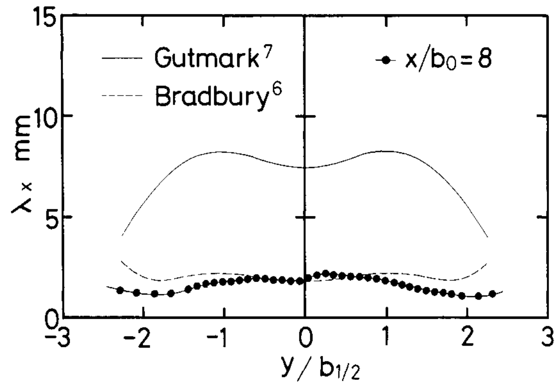

Fig. 12 Profile of λ_x at $x/b_0 = 8$.

region, the profile of $\Lambda_{x\tau}$ possesses the characteristics of the simple shear layer. At $x/b_0 = 8$, however, the profile has only one minimum value in the central portion of the jet and is similar to Gutmark's result as shown in Fig. 10. It is seen that, with regard to the profile of $\Lambda_{x\tau}$, the characteristics of the simple shear layer disappear very rapidly downstream of the initial mixing region. Figures 9 and 10 also show that the integral scale is enlarged in the outer portion of the curved jet and reduced in the inner portion of it, due to the influence of the centrifugal forces.

Figures 11 and 12 show the profiles of λ_x at $x/b_0 = 3$ and 8, respectively, and compared with the results for a simple shear layer and a two-dimensional straight jet. For both the simple shear layer and the two-dimensional jet, the value of λ_x far downstream of the nozzle increases almost linearly with x.[7,8] However, the hypothetical origin of λ_x is significantly far from $x = 0$, and λ_x grows gradually in the self-preserving state. Therefore, λ_x shown in Figs. 11 and 12 is not made dimensionless by multiplying $1/x$. The profile at $x/b_0 = 3$ has a sharp maximum at the central portion of the jet. In contrast with $\Lambda_{x\tau}$, the profile of λ_x in the initial mixing region of the jet is very different from that of the simple shear layer measured at

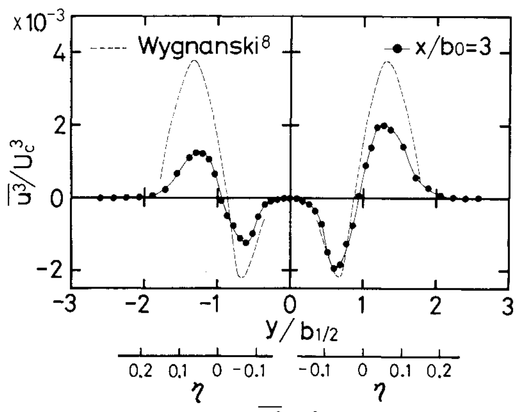

Fig. 13 Profile of $\overline{u^3}/U_c^3$ at $x/b_0 = 3$.

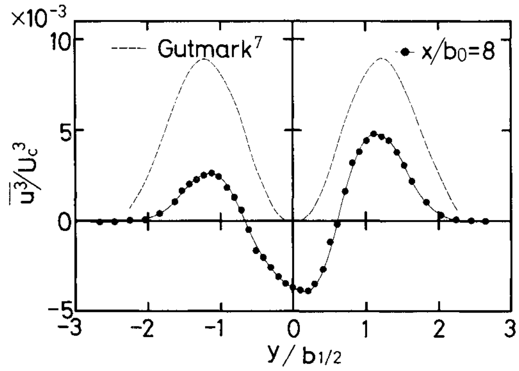

Fig. 14 Profile of $\overline{u^3}/U_c^3$ at $x/b_0 = 8$.

$x \doteqdot 500$ mm, the reason being that the turbulent intensity $\overline{u^2}$ that appears in Eq. (3) develops rapidly in the potential core and the dissipation in the potential core is very small as shown in Fig. 13 of the previous paper.[5] The profile of λ_x at $x/b_0 = 8$ is fairly similar to those obtained by Gutmark et al.[7] and Bradbury,[6] although the measured values of the present study are very small compared to Gutmark's results, which were measured far downstream of the nozzle ($x/b_0 \doteqdot 100$). Bradbury's results agree fairly well with the present experiment. Bradbury's results are considerably smaller compared to Gutmark's results; the reason may be that Bradbury's results were obtained about 50 slot thicknesses downstream of the nozzle exit and Bradbury studied the structure of a jet exhausting into a weak external stream. It can be pointed out from the present study that no significant asymmetry is observed in the profiles of λ_x as shown in Figs. 11 and 12.

The skewness factor S and the flatness factor F are defined as follows:

$$S = \overline{u^3}/(\overline{u^2})^{3/2} \quad (4)$$

$$F = \overline{u^4}/(\overline{u^2})^2 \quad (5)$$

Figures 13 and 14 show the profiles of $\overline{u^3}/U_c^3$ at $x/b_0 = 3$ and 8, respectively. The profile of $\overline{u^3}/U_c^3$ at $x/b_0 = 3$ agrees qualitatively well with Wygnanski's results for a self-preserving simple shear layer. However, the values of the present study are considerably lower than Wygnanski's results at the extremes of the jet, since Wygnanski's results were obtained far downstream of the nozzle. On the other hand, the absolute values of present results are relatively large in the central portion of the jet and agree quantitatively fairly well with Wygnanski's results, because the turbulence develops rapidly in the potential core. The profile of $\overline{u^3}/U_c^3$ at $x/b_0 = 8$ agrees qualitatively well with Gutmark's results. The profile has a minimum near the jet centerline and a maximum in each side of the jet. However, the levels of Gutmark's results are higher than the

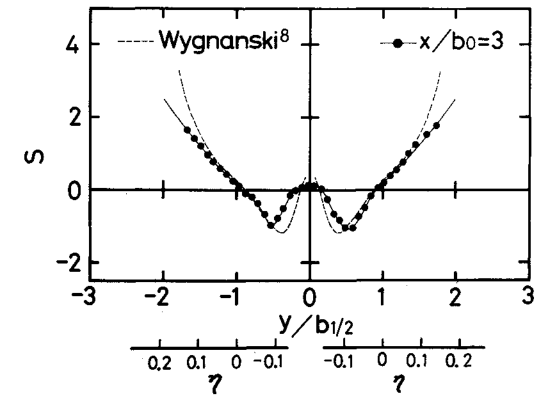

Fig. 15 Profile of S at $x/b_0 = 3$.

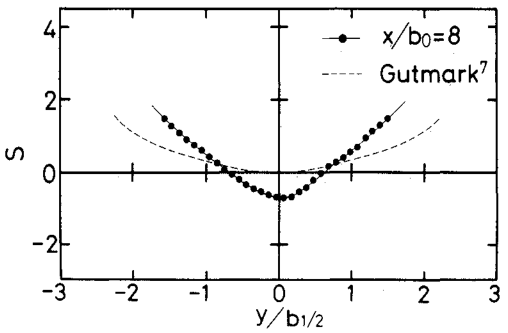

Fig. 16 Profile of S at $x/b_0 = 8$.

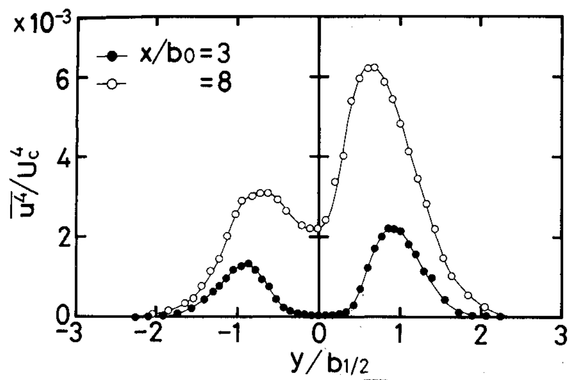

Fig. 17 Profile of $\overline{u^4}/U_c^4$.

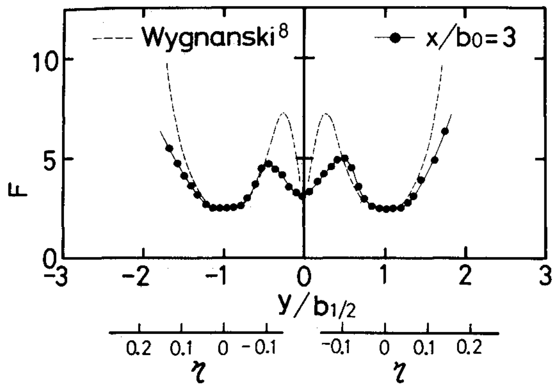

Fig. 18 Profile of F at $x/b_0 = 3$.

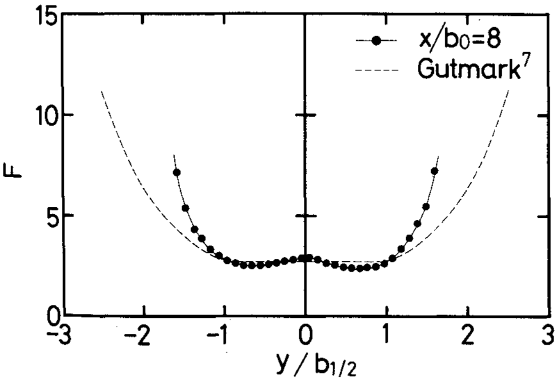

Fig. 19 Profile of F at $x/b_0 = 8$.

present results, since Gutmark's results were obtained far downstream of the nozzle. In contrast with Gutmark's results, the values of $\overline{u^3}/U_c^3$ of the present results are negative in the central portion of the jet. This trend is also observed in the profile at $x/b_0 = 3$. Therefore, it is thought that the characteristics of the simple shear layer are still preserved at $x/b_0 = 8$. In Figs. 13 and 14, it is seen that $\overline{u^3}/U_c^3$ is clearly enhanced in the outer portion of the curved jet and reduced in the inner portion of it.

The skewness factor is calculated using the data on $\overline{u^3}/U_c^3$ shown in Figs. 13 and 14, and the data on $\overline{u^2}$ shown in the previous paper.[5] Figures 15 and 16 show the profiles of S at $x/b_0 = 3$ and 8, respectively. The profile of S at $x/b_0 = 3$ agrees well with Wygnanski's results. The profile at $x/b_0 = 8$ agrees qualitatively well with Gutmark's results, although S is negative in the central portion of the jet. As seen from Figs. 15 and 16, the effects of the centrifugal forces do not lead to the asymmetry in the profile of S.

In Fig. 17 are shown the profiles of $\overline{u^4}/U_c^4$ at $x/b_0 = 3$ and 8. The profiles clearly show the asymmetry. It is seen that $\overline{u^4}/U_c^4$ is enhanced in the outer portion of the curved jet and reduced in the inner portion of it due to the influence of centrifugal forces. Figures 18 and 19 show the profiles of F at $x/b_0 = 3$ and 8, respectively. The profile of F at $x/b_0 = 3$ agrees well with Wygnanski's results for a simple shear layer, and the profile at $x/b_0 = 8$ agrees fairly well with Gutmark's results for a two-dimensional straight jet. It is also seen that the effects of the centrifugal forces lead to little asymmetry in the profile of F.

Conclusions

The experimental results of the present investigation on the initial mixing region of a two-dimensional curved jet lead to the following conclusions.

1) The values of $y_{\gamma=0.5}/b_{1/2}$ are nearly one at the nozzle exit and grow very rapidly in the initial mixing region. However, downstream of the initial mixing region, they decrease gradually and approach a value for a self-preserving, two-dimensional straight jet. The extent of the turbulent zone in the outer portion of the curved jet is larger than that in the inner portion due to the influence of centrifugal forces. This tendency is significant in the initial mixing region but it decays gradually with the downstream distance.

2) The profile of $\Lambda_{x\tau}$ at $x/b_0 = 3$ possesses a characteristic of the self-preserving simple shear layer, and the profile at $x/b_0 = 8$ is similar to the profiles of the self-preserving, two-dimensional straight jet. In contrast with $\Lambda_{x\tau}$, the profile of λ_x at $x/b_0 = 3$ is very different from the simple shear layer, although the profile at $x/b_0 = 8$ is quite similar to that of the self-preserving, two-dimensional straight jet. The integral scale is enlarged in the outer portion of the curved jet and reduced in the inner portion of it by the curvature effects. However, very little asymmetry is observed in the profile of λ_x.

3) The profiles of S and F at $x/b_0 = 3$ and 8 agree fairly well with the corresponding profiles of the self-preserving simple shear layer and the self-preserving, two-dimensional straight jet, respectively. However, the profile of S at $x/b_0 = 8$ shows that the characteristics of the simple shear layer are still preserved, since the values of S at $x/b_0 = 8$ are negative in the

central portion of the jet. The profiles of S and F are almost symmetric. Therefore, it is thought that the effects of centrifugal forces on the probability density distribution of the fluctuating velocity are negligibly weak.

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June 26, 1989

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Duties of the Editor-in-Chief encompass the following: Foreseeing and stimulating major contributions to the journal, with assistance from an Editorial Advisory Board. Logging in, acknowledging, and appraising submitted manuscripts; checking their general quality, importance to the technical community, and compliance with editorial specifications; assigning them to Associate Editors for processing; arbitrating editorial disputes; tracking manuscripts and Associate Editor actions via computer.

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Address questions about editing procedures or other factors connected with duties to: Norma Brennan, AIAA Director, Editorial and Production Departments (202-646-7482).

Questions concerning policy may be directed to me in writing at the following address: Dr. Billy M. McCormac, AIAA Vice President – Publications, D91-01/B256, Lockheed R&DD, 3251 Hanover Street, Palo Alto, CA 94304

B. M. M.